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Contributions of Operations Research to Location Theory

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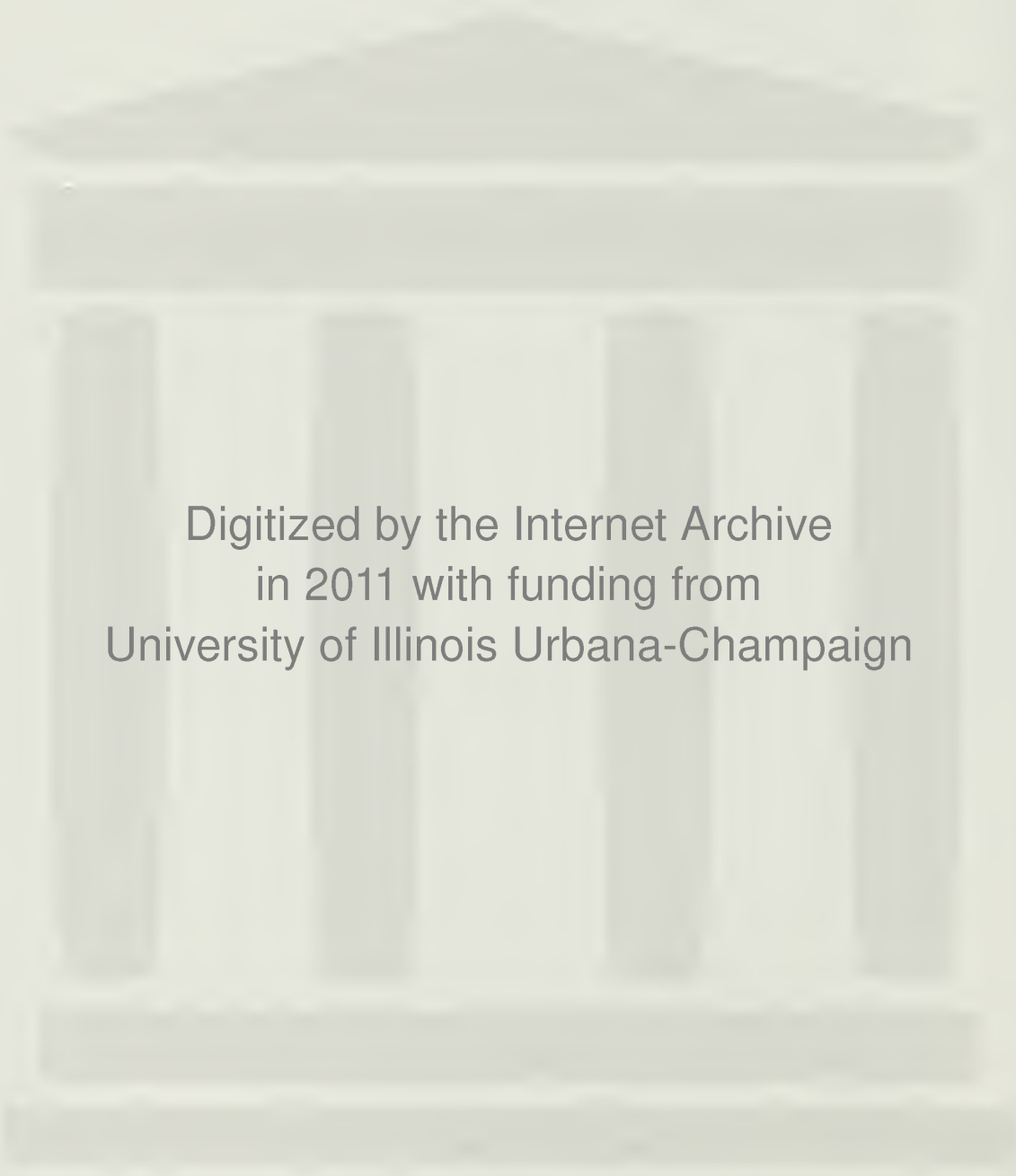
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Abstract

Location problems which can be quantified as optimization problems are natural candidates for operations research approaches, and many such problems have been studied using mathematical programming methodology in the last 30 years or so. This paper attempts to give a largely non-technical overview of some of this activity. A number of actual problems are discussed, and then models of these problems are presented. Models are classified as planar, network, and mixed integer programming models, and methodology for solving such types of models is outlined. A particularly important location problem, known as the simple plant location problem, or warehouse location problem, is discussed in some detail, with the emphasis placed on solution approaches. The three model classes are compared in terms of four important attributes: realism, data requirements, computational requirements, and difficulty of explanation; no single class is best in all four attributes. Some opportunities for future work, particularly the need for generally available software, are identified. References are given for further reading, including texts and review papers.

1. Introduction

Most of the contributions of Operations Research/Management Science (OR/MS) to location theory have occurred within the last 30 years. This period constitutes about 60% of the lifetime of the profession itself, which is little more than 50 years old. We think there is little doubt that the contributions consist principally of algorithms - well-defined computational procedures for solving quantifiable problems. These algorithms have built largely upon results in an area known as mathematical programming, which got its start soon after World War II with the now famous simplex method of George Dantzig (see Dantzig, 1949) for solving linear programming problems.

It is important to realize that a location problem must be quantifiable in order for there to be any hope of solving it with an algorithm: there must be a well-defined objective to be optimized, e.g., cost to be minimized, or profit to be maximized. Likewise there are usually well-defined constraints, e.g., budget constraints, which limit the scope of the optimization. Location problems which are highly subjective or political in nature are thus usually not very good candidates for operations research approaches, although even for such problems there may be results which can help to reduce the scope of the problem under consideration, or identify basic tradeoffs of interest.

Many of the best known OR algorithms for solving location problems involve choosing best locations from a finite collection of possible sites. Since a site either is chosen or is not, such problems are intrinsically discrete in nature, and are candidates for being solved as integer programming problems. Suppose, for example, a large banking corporation is planning to move into a major metropolitan area; they wish to choose 10 locations from 60 possible sites under consideration. Numbers like 10 and 60 are familiar to everyone, and a layperson might well think that the problem of the corporation is not too difficult; a computer could just enumerate all the possibilities and choose a best one. However, the well-known combination formula tells us that there are $C(60,10) = 60!/(10!50!)$ possible choices of 10 locations from 60 possible sites. A computer that could enumerate 10,000 choices per second while operating 24 hours per day would require about 87.26 days (with no time off for week ends) to make a best choice. If, instead of 60 possible sites, there were either 70, 80, 90, or 100, this same computer would need, respectively, 1.26 years, 5.22 years, 18.14 years, and 54.89 years, to make a best choice. Even with today's advances in computers, there is little hope that total enumeration will be useful! Fortunately, algorithms can be used to greatly reduce the amount of computational effort needed to solve such problems. Even then, it must be admitted, for a sufficiently large problem, that an algorithm may not be able to provide an optimal

solution. It may be necessary to be satisfied instead with one which is, say, within 1% of optimality.

To continue with our example, the corporation might instead be uncertain how many branch banks there should be. The corporation would realize that the more banks it locates, the more convenient the branches would be to its customers in terms of travel time or travel cost. On the other hand, the more branches there are, the higher would be the operating expenses and fixed costs. Thus there is a tradeoff between convenience and operating costs, which it would be important to analyze. Such tradeoffs often occur in solving location problems. As another example, consider a nation-wide corporation mailing out monthly bills. The corporation can specify the addresses to which customers will send their checks. These addresses could be branch offices, or possibly banks which would collect and process the payments for a fee. While the checks are in the mail, the corporation of course earns no interest on the money. Since the amount of money involved is substantial, the corporation would of course like to get the money quickly. In fact, it has a trade-off to analyze; interest obtained on the payments versus the costs of the branch offices or charges of the banks for processing the payments. This particular problem has come to be known as the lockbox problem, and has received substantial attention. We devote a section of this paper to a related problem: the simple plant location problem. A little contemplation should make it clear that a government choosing post office locations, or a national distributor choosing sites for retail stores, faces related problems.

When a location problem has substantial transport costs, but no fixed site costs of importance, there are several other approaches to modeling it. Often it is assumed that transport costs are directly proportional to transport distances. When these distances are incurred on a transport network, such as a road network, the result is often a network model. Such models usually employ shortest path algorithms to compute travel distances. The focus of network model research has been principally upon two topics: 1) algorithms to solve the problems, and 2) localization results, such as vertex-optimality results, which reduce to a finite collection the set of locations which must be considered to obtain an optimal solution. Once such a finite set is obtained, the resulting remaining problem may well be modeled as an integer or mixed integer programming problem.

It is possible, for a network location problem, that it may be prohibitively expensive to obtain, or to work with, the necessary network data. In such cases network distances may well be approximated using planar distances, e.g., Euclidean or rectilinear distances. The resulting problems are often easier to analyze, and can be helpful for

providing insight. In this case we are dealing with what we call a planar model. Often results from nonlinear programming can be employed to help solve such a model.

In the coming sections we consider planar models, network models, and mixed integer programming models. This classification of models is not exhaustive, but it does capture much of what has been dealt with in the literature. Needless to say, our discussion to follow reflects our own interests and tastes, and we do not claim to have identified all the important contributions of OR to location theory. Many painful omissions have been necessary, and we hope these omissions will not cause offense. For more detailed reading in the area with which our paper deals, the reader is referred to texts by Handler and Mirchandani (1979); Love, Morris and Wesolowsky (1988); Mirchandani and Francis (1990); and Francis, McGinnis and White (1992). Also, as additional sources, we list in our references, arranged in a separate section, a number of survey papers.

The rest of the paper is organized as follows: In section 2 we present some examples of location analysis. We then introduce mathematical models of these examples. We close section 2 by providing a brief review of contributions made by operations research in solving these models. To provide a glimpse of various kinds of techniques used by operations researchers, we focus on a single model (the simple plant location model) in detail in section 3. A summary of the paper and some concluding remarks are given in section 4.

2. Selected Examples, Models, and Contributions

In this section we first present several examples of actual situations where operations research-based techniques have made contributions to location decision making. We then present some of the basic but popular and useful models of location theory and show how each example is modeled. This list is not exhaustive, but is intended to provide some feel for what is available in the literature. After discussing the models, we briefly discuss the solution approaches available for these models. In presenting the solution methods, we look explicitly at the "problem space" used in the model. Classification based on the problem space is useful because the types of solution methods, range of model applicability, and the ease of solution tends to depend on the nature of the problem space.

2.1 Application Examples

These examples represent actual applications. In some cases we have simplified

the problem to focus on the main issues involved. For an exact description of the application, solution procedure used and the impact of the study, please see the original publications.

Example 1. Center Problem (Patel, 1981)

Service centers were to be located in Dharampur taluka (corresponding to a county in the US) in the state of Gujrat in India as a part of a rural development plan. These service centers housed offices of various functionaries such as school teachers, health center workers and post office workers. The aim of the project was to enhance the availability of functionaries to the rural population of Dharampur and to provide all villages with reasonable access to health, economic and educational service. To this end, initially it was decided that service centers should be located in the taluka in such a way that every village would have a center within a five mile radius.

A preliminary analysis led to a selection of 44 out of the 237 villages as candidate sites for service centers with an estimated cost of Rs. 5.5 million for building service centers at these 44 sites. However, the budget for this task was Rs. 1.4 million. Thus, in order to decide how to reduce the number of service centers from 44 to meet the budget, a seminar was organized with local leaders, voluntary workers, government officials and planners. This, however did not lead to any conclusive decision since each party was interested in their own views on where and why the service centers should be chosen. Therefore, the planners decided to establish an objective criterion and used operations research techniques to select the locations (an exhaustive enumeration would evaluate $2^{44}=1.76 \times 10^{13}$ possibilities). The criterion chosen was to minimize the maximum distance any person has to travel to reach the nearest service station.

Example 2. Covering Problem (Plane and Hendrick, 1977)

A comprehensive system-wide study of fire company locations of the city of Denver was carried out. The response time of fire-suppression vehicles was used as a surrogate performance measure of fire protection. A maximum permissible response time to any point in the city, and the average, along with the distribution of the response time, were considered to be important to judge the level of service. Since the majority of the cost of the fire department is proportional to the number of fire stations, minimizing the number of fire stations while maintaining the service level was desired.

Example 3. Partial Covering Problem (Eaton et al., 1985)

In 1974-75, the city of Austin, Texas, decided to operate its own emergency

medical service (EMS) department instead of contracting this task to an outside company. In order to determine the number and location of vehicles to deploy to provide quality service at a reasonable cost, the city was broken into 358 zones. Travel times, developed by the Austin Transportation Study Office, were used as EMS response times.

The EMS official wanted to have a goal which exceeded the performance measure of the 1973 EMS Act of covering 95% of all demand within 10 minutes driving time. However, the estimation of demand was not clear. Several surrogates of demand, such as total population, were recognized. In addition, the historical distribution of calls for EMS service based on geographic region was available. A system was desired that was easy to understand and inexpensive, since it had to be run several times to address the implications of different options.

The multiobjective nature of the decision called for maximization of the system performance on these demand surrogates as well as minimization of the number of vehicles deployed.

Example 4. "Warehouse" Location (Fitzsimmons and Allen, 1983)

The comptroller of Public Accounts in the State of Texas uses tax audits to increase compliance with tax laws and enhance collection of sales taxes and other taxes. Until 1980, all auditors responsible for out-of-state corporations were based in Austin and traveled from Austin to out-of-state corporations. These visits were typically two weeks long and the auditors spent one half of their time on trips. This contributed to high turnover of auditors.

A decision was made to open audit offices at out-of-state locations in order to reduce travel expenses, reduce auditor turnover, encourage voluntary compliance and provide improved taxpayer service. Although any site in the continental US could have been chosen, a list of 12 potential cities was developed. From this list it was to be decided how many offices should be opened, which cities should be selected, what states would be served by each office and how many auditors should be assigned to each office.

Example 5. Multifacility Location (Love and Yerex, 1976)

The Coastal Construction Company (the name has been disguised) had three existing facilities (and two additional buildings on its premises) where it produced components for large bridges and tunnels. A new product, transmission poles for electrical utilities, was to be introduced and the location of two new production facilities was to be determined. The three existing facilities were: a concrete batching plant, steel manufacturing, and a storage and shipping area. Two new facilities, a concrete casting

area, and an assembly and storage area, were to be located in the available space.

The new concrete casting area will require material from the concrete batching plant and the steel manufacturing and storage area. A crane is used to move the cast parts from the concrete casting area to the assembly and storage area. All completed products are shipped through the shipping area. Thus the two new facilities had material flow between them as well as with the three existing facilities. The objective was to determine the location of these two new facilities such that total material handling cost is minimized.

Assuming rectilinear travel distance, the problem was formulated and solved as a linear programming problem, and the solution was implemented.

Example 6. Median Problem (Cerveny, 1980)

Two regional centers of the American Blood Program had the option of changing their main staging area configurations in the near future and therefore wanted to examine the effects of such a change. A staging area is where bloodmobiles are provisioned, repaired, and kept between blood collections. Mobile vehicles travel to the collection sites to collect donated blood, and return to the staging area. Each of the regional centers wanted to find the best location for a central staging area in order to minimize the distance traveled. They also wanted to explore the possibility of a central site with sub-sites.

Example 7. Quadratic Assignment Problem (Dickey and Hopkins, 1972)

Locations of several new buildings was to be determined on the campus of a university. There were several available sites and the new buildings were to be installed at these sites. Each building was to be used for a specific purpose, e.g., lecture hall, laboratory, offices. The estimated number of people that will be traveling between pairs of buildings was available and the distances between every pair of sites was also available. The objective was to find the assignment of each building to a site such that the total amount of required travel is minimized.

In each of the above applications, the site selection was done by first developing a location model of the system and then analyzing and solving the model. A location model is a mathematical model representing the relationships between the key elements of the location decision factors. The aim of a location model is to provide a decision aid in choosing new facility locations.

2.2 Basic Models

We will next give examples of basic models used in analyzing location problems. We will also indicate how each of the above examples can be formulated as one of these models. In developing the models we shall use the following notation:

$P_1, \dots, P_m$: given points in the plane, demand points or "existing facility" locations

X : location of a single new facility

$d(X, P_i)$: appropriate distance between X and P_i

$X_1, \dots, X_n$: locations of n new facilities

$\underline{X} = (X_1, \dots, X_n)$

$D(\underline{X}, P_i) = \min\{d(X_j, P_i) : j = 1, \dots, n\}$: distance between P_i and the nearest new facility.

Model 1: The Center Problem

Consider the example of locating service facilities in India. The objective that the planners chose reflected a desirability of equity, i.e., - minimizing the maximum distance a person has to travel. Let $g(\underline{X}) = \max \{D(\underline{X}, P_i) : i=1, \dots, m\}$ represent the maximum distance any person has to travel; then the problem can be posed as

$$\text{Min}_{\underline{X}} g(\underline{X}).$$

This problem is known as the *n-Center problem* and, besides other applications, has been used to model locations of emergency medical facilities, location of a helicopter to minimize the maximum time to respond to an emergency, and location of a transmitter to maximize the lowest signal level received.

Model 2: The Covering Problem

Suppose that in the example of problem 1, the planners wanted to determine the minimum investment needed to build enough service stations so that there is a station within 5.5 miles of every village. Suppose the 44 selected sites are the candidate sites for the location of service centers. For each customer (village here) and each potential location site j (the 44 sites here) define

$$a_{ij} = \begin{cases} 1 & \text{if customer } i \text{ is within the required distance of site } j \\ 0 & \text{otherwise} \end{cases}.$$

Let c_j denote the cost of building a service center at site j . The problem can be modeled as the following mathematical program, known as the *set covering problem*:

$$\begin{aligned} & \text{Min } \sum_j c_j x_j \\ & \text{Subject to: } \sum_j a_{ij} x_j \geq 1, \quad \text{for all } i \\ & \quad x_j \in \{0, 1\}, \end{aligned}$$

where $x_j = \begin{cases} 1 & \text{if a server is located at site } j \\ 0 & \text{otherwise.} \end{cases}$

The problem of determining the minimum number of fire stations (example 2) while maintaining a given level of response time can be similarly modeled as a set covering problem.

Model 3: Partial Covering Problem

In example 3, if we choose any surrogate of demand, say total population, then the aim of the planners would be to find the minimum number and locations of facilities which can cover 95% of the population within 10 minutes driving time. For a given number of facilities, the problem then is to maximize the number of demand points that can be covered within a specified radius (called covering radius, which is 10 minutes here). This problem is called the *partial covering problem*.

In the case of example 2, a series of partial covering problems with different values of the covering radius gives a solution to the original problem for any chosen surrogate of demand. Let a_i denote the population in customer zone i and N_i the set of zones that are within the required time of zone i . Variable $y_i = 1$ if zone i is covered and 0 otherwise, and variable x_j is as defined in model 2. The partial covering model is then given by:

$$\begin{aligned} & \text{Max } \sum_i a_i y_i \\ & \text{Subject to: } \sum_j x_j = P \\ & \quad \sum_{j \in N_i} x_j - y_i \geq 0 \\ & \quad x_j, y_j \in \{0, 1\}, \end{aligned}$$

where P is the number of new facilities to be located.

Model 4: Warehouse Location Problem

In the warehouse location problem (WLP), we want to open a number of warehouses (the actual number is a decision variable) to serve a given set of customers.

We are given a set S of candidate sites where warehouses can be built and suppose that it costs f_j to build a warehouse at site $j \in S$. The cost of serving a demand point i per unit distance is given by w_i . The objective is to minimize the sum of the fixed and variable costs of serving the demand points. With x_j defined as in model 2, the problem can be written algebraically as

$$\text{Min}_{\underline{X}} \sum_j f_j x_j + \sum_i w_i D(\underline{X}, P_i).$$

The problem of locating tax audit offices in example 4 can also be modeled as a WLP. Here the analogous terms between the tax-audit problem and the warehouse location problem are: (Warehouse-Audit Office; Facility Fixed Cost-Office Rent and Overhead; Customer Demand Zone-State; Commodity Demand-Audit Trips; Transportation Cost - Air Fare plus per Diem). An extension of WLP is when there is a capacity restriction on each warehouse. To illustrate this, in example 4 the maximum number of audit hours available can be the capacity of an audit office. However, in this particular application, a warehouse location model (not the capacitated version) was used. The lockbox problem discussed in the introduction is also an example of a problem that can be modeled as a WLP.

Model 5: Multifacility Minisum Problem

In the case of Coastal Construction Company (example 5), there is a fixed amount of interaction cost per unit distance between new facility j and existing facility i . In addition there are interactions between pairs of new facilities, the cost of which is a function of weighted distance between them. This problem can be written as:

$$\text{Min } f(\underline{X}) = \sum_i \sum_j w_{ij} d(X_j, P_i) + \sum_j \sum_k v_{jk} d(X_j, X_k),$$

where w_{ij} is the interaction cost per unit distance between existing facility i and new facility j ; v_{jk} is the interaction cost per unit distance between new facilities j and k . The w_{ij} and v_{jk} often depend on the means by which transport between facilities occurs, e.g., on the type of material handling equipment used.

The advancement in transportation and communication technologies and the globalization of business has forced many firms to adopt a multifacility production strategy. This model captures a particular location problem faced by a firm following such a multifacility production strategy. For example, the firm may want to locate m

production plants, each producing some end products as well as several components or by-products which are used by other plants. The existing facilities may be the customer locations or the distribution centers where the customers' demand for the products produced by each plant are known.

Model 6: n-Median Problem

In the *n-median problem*, n new facilities are to be located. Existing customers choose the new site that is closest for service. The objective is to minimize the sum of the weighted transportation cost incurred in serving the customers. Mathematically this can be written as:

$$\begin{aligned} \text{Min } f(\underline{X}) &= \sum_i w_i D(\underline{X}, P_i) \\ \text{Subject to: } |\underline{X}| &= n, \end{aligned}$$

where w_i is the weight (relative importance - perhaps population) of customer location i , and $|\underline{X}|$ denotes the number of elements in $\underline{X}$.

The bloodmobile location problem of example 6 is an instance of the 1-median problem when only a single central staging area is to be located and is the n -median problem when a central staging area as well as $(n-1)$ subsites are to be located.

Model 7: Quadratic Assignment Problem

The quadratic assignment model has many diverse applications including: in-plant layout, machine scheduling, keyboard assignment for a typewriter, and analyzing chemical reactions for organic compounds. In the example of campus planning (example 7 above), there are n available sites and n new facilities are to be installed at these n sites. Let $\{b_{ij}\}$ be a matrix of estimated number of students and staff that will be traveling between facilities i and j . Let the distances between any pair of sites, say k and l , be given by a matrix $\{a_{kl}\}$. The objective is to find the assignment of each facility to a site such that the total travel distance incurred by all the people is minimized. Note that at each site one and only one facility can be located. The important feature of the model is that the total travel distance for every pair of facilities (number of people traveling times the distance traveled) depends on the location of *both* of the facilities.

To state the model mathematically, define variables x_{ik} equal to 1 if facility i is assigned to site k , and 0 otherwise.

$$\begin{aligned}
& \text{Min } \sum_i \sum_j \sum_k \sum_l a_{kl} b_{ij} x_{ik} x_{jl} \\
& \text{Subject to: } \sum_i x_{ik} = 1, \quad \text{for all } k, \\
& \quad \quad \quad \sum_k x_{ik} = 1 \quad \text{for all } i.
\end{aligned}$$

2.3 Solving the Models

In order to describe selected contributions that operations research has made towards providing solution procedures for the above problems, we divide the problem space into three classes: planar models, network models, and discrete models. Many of the above models can be posed on any one of the three spaces with some slight modifications, and almost all models can be put into one of the three classes. In our discussion we will refer to the demand points as *existing facilities* and the facilities to be located as *new facilities*.

The main difference in these three classes of models is the manner in which the distance between two points is defined. In planar models, the distance function $d()$ is a "norm", often Euclidean, rectilinear, or some other norm, and the number of possible locations for new facilities is infinite. This renders the corresponding problems as continuous. If (x_i, y_i) are the coordinates of a point i , then the Euclidean distance between points i and j is given by $\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$, while the rectilinear distance is given by $\|x_i - x_j\| + \|y_i - y_j\|$, where $\|\cdot\|$ is the absolute value function.

In network models we have a transport network on which travel occurs. The transport network, for example, may represent a system of major highways and/or roads. The distance between two points is usually defined as the shortest distance on the network. Distances are often more accurately represented in network models than in planar models, but the need for data is also higher in network models since the length of each segment is needed. For many models it becomes advantageous to work directly with the network, exploiting its properties in developing a solution procedure. The existing facilities are located on the nodes of the network, and the new facilities are to be located at points on the network. An additional advantage of network models is that they make problem visualization easier. Thus even if the problem is not solved as a network problem, a solution presented in network form may assist the decision maker in his/her understanding of the problem and the issues involved.

In discrete models, distances may be derived from planar or network distances, or some more general type of transport cost which is proportional to distance. The number of existing facilities and the number of potential new facilities is finite. Discrete problems are often modeled as mixed integer programs and are generally more difficult to solve.

On the other hand, many realistic assumptions can be incorporated in discrete models which cannot be included in planar or network models.

A solution technique can be classified as an *exact method* or a *heuristic method*. An exact method finds an optimal solution whereas a heuristic method finds only an *approximately optimal solution*. The solution provided by a heuristic method is approximate in the sense that it is not guaranteed to be an optimal solution, and although it may be very close to the optimal solution, just how close it is often cannot be known with certainty.

Besides the solution quality, another important attribute of a heuristic is solution "time". The time can be given in terms of the average-case time or the worst-case time. The average-time is usually determined empirically. The worst-case time (also known as the complexity order) of an algorithm or solution procedure (heuristic or exact) is usually stated in terms of the size of an instance of the problem, and measures the maximum total number of steps (amount of work) that it takes to solve the problem. If an algorithm is of *polynomial time*, it means that if, say, both m and n represent the size of an instance of the problem, then the total effort (work) to solve the problem will be less than or equal to some polynomial function of m and n .

Many location problems are either non-convex and/or fall in the class of NP-hard problems. If a problem is NP-hard, it means that there is little hope of finding an algorithm that has its worst-case running time bounded by a polynomial function of the size of the problem (Garey and Johnson, 1979). This makes the search for an exact solution procedure a futile one, especially if the problem is large. If an optimization problem is convex, a desirable property is that a local minimum is also a global minimum. This fact often leads to efficient procedures to solve convex programming problems. A contribution of Operations Research/Management Science has been in providing exact methods for some problems and heuristic solution procedures for inherently difficult problems. In addition, some research has involved identifying and solving special classes of otherwise difficult problems. These special cases help in providing insight into the more general version of the problem and can be useful as building blocks for devising procedures for the general problem. Some selected contributions for each of the models discussed above now follows. In this discussion, we omit the simple plant location model, as it is treated in detail in section 3.

2.3.1. Planar Models

1-Center Problem

The n -Center problem with $n=1$ (a single new facility to be located), $w_i = 1$ for all i , and Euclidean distances is equivalent to finding a circle of smallest radius containing the m existing facility locations. There are a number of very efficient and simple algorithms to solve the problem, and an excellent survey is given in Hearn and Vijay (1982). With the advent of computer graphics and fast computers, an interactive human-machine approach is very attractive (see Brady and Rosenthal, 1980): Draw m circles of radius r with the m existing facilities as centers. Beginning with a very small value of r , increase r until the intersection of all m circles is a single point. This point is the solution to the circle covering problem.

When all of the weights are equal (1 or any other number), the problem is called *unweighted*, otherwise it is *weighted*. The weighted problem can still be solved using the above human-machine approach. In this case the radius of the circle to be drawn about a point i is r/w_i . We note that the 1-center problem is a convex programming problem, and there are other algorithms to solve it that exploit its special structure.

1-Median Problem

When the distance is rectilinear, the problem decomposes into two independent subproblems, one for the x -coordinate and the other for the y -coordinate of the new facility. The solution procedure exploits the following two properties of an optimal solution: (i) the x -coordinate will coincide with the x -coordinate of some existing facility (similarly for the y -coordinate) and (ii) no more than half of the total weight can be on either "side" of the optimal facility location. The latter property is also known as the *median condition*. Let W denote the sum of the weights of all of the existing facilities. In order to find the optimal x coordinate, make a list of the x coordinates of the existing facilities and arrange them in an increasing order. Starting with the first coordinate (the lowest value), move up the list until the sum of the weights of the existing facilities encountered first totals at least half of W . This gives the optimal x coordinate. The optimal y coordinate is similarly found working from bottom to top.

When the distance is Euclidean, the usual solution approach is via an iterative algorithm given by Weiszfeld (Francis, McGinnis and White, 1992). At step k , given a location (x^k, y^k) of the new facility, the algorithm gives an improved location (x^{k+1}, y^{k+1}) by using a simple formula. This formula is derived by setting the partial derivatives of the objective function to zero and deriving expressions for updated values of the x and y coordinates of the new facilities. The procedure continues until no appreciable

improvement in the objective function is found.

Multifacility Minisum Problem

For rectilinear distances the problem separates into two independent subproblems, one for each coordinate, and each of these can be solved with linear programming or network flow algorithms. We will soon describe another method while discussing this problem as a network model, which can be used to solve each subproblem. For Euclidean distances, a generalization of Weiszfeld's algorithm is used.

Multifacility Minimax Problem

The Euclidean distance problem can be solved by a procedure which is in the spirit of Weiszfeld's algorithm (Love, Morris, and Wesolowsky, 1988). For the rectilinear distance problem, using a linear transformation, the problem can be solved by solving two independent linear programming problems (Dearing and Francis, 1974).

n-Median Problem

For both rectilinear and Euclidean distances, the problem is non-convex and is difficult to solve. The alternate location-allocation heuristic of Cooper (1963) is widely used. The heuristic starts with an initial location of new facilities and allocates the existing facilities to the nearest new facility. Given the set of existing facilities allocated to a new facility j , the best location of j is found by solving a 1-median problem. This relocation is performed for each new facility. Again, new allocations are found and the procedure is repeated until no improvement in the objective function value is found. Other methods discretize the set of possible new facility locations and use a mixed-integer programming approach which we describe later.

2.3.2 Network Models

Location models usually involve transportation on a network, e.g., roads, rivers, and flight corridors. Planar models, discussed above, are often easier to solve but are often approximations of network models. Network models work directly with the network and assume that the new facilities are to be located on the network, which approximates the desire in many real situations to locate facilities close to the transportation structure. In addition, these models assume that all of the existing facilities are located on the network as well. The reason to work with the network explicitly is to be able to exploit the structure of the network.

A network N consists of a collection of *nodes* and *arcs* where each arc has a pair

of nodes as its end points. Two arcs are *adjacent* if they share a common node. Physically, if the network represents a road structure, the nodes are usually points where there is a "crossover" of two or more roads. In addition, points where demand occurs and a set of candidate location sites (if known and finite) are also represented as nodes. Each arc has an associated *positive length*. A *path* is a sequence of adjacent arcs, where *adjacent arcs* share a common node. In the models formulated above, we simply need to interpret the distance function $d(x,y)$ to be the length of a shortest path between points x and y on the network. A special kind of network that has received close attention is *tree*. A tree with n nodes consists of $(n-1)$ arcs such that there is a path between every pair of nodes. An important property of tree networks is the convexity of distance on the tree (Dearing, Francis and Lowe, 1976).

An important class of results for many location problems involves the identification of a *Finite Dominating Set* or *FDS* (Hooker, Garfinkel and Chen, 1991). A FDS for a problem is a finite set of points such that there is at least one optimal solution consisting of points in the set. The identification of a FDS is a very useful result in analyzing a location problem on a network because it helps in modeling the continuous problem as a combinatorial problem. Thus, instead of dealing with infinitely many possible location sites, the search for an optimal location can be restricted to a finite (hopefully small) list of potential sites.

n-Center and Covering Problems

For the n -center problem we define a *bottleneck* point Y_{ij} to be a point on the network for which $w_i d(Y_{ij}, P_i) = w_j d(Y_{ij}, P_j)$ for some distinct i and j . The set of bottleneck points along with the set of nodes of the network constitutes the FDS for the n -center problem. Thus the 1-center problem can be solved by evaluating the objective function at each point of the FDS and choosing a point which minimizes it. When the network is a tree, the problem is a convex optimization problem. Handler (1973), and Kariv and Hakimi (1979) give algorithms that can solve the problem efficiently.

A substantial literature exists on solution methods for the n -center problem. Most of this research exploits the existence of a FDS. A number of approaches, e.g. Minieka (1970), rely on solving a sequence of covering problems to solve the n -center problem. Recall that in a covering problem the minimum number of new facilities to serve the existing facilities within a specified distance r , is sought. The distance r is called the *covering radius*. Suppose we have a solution to the covering problem with covering radius r that locates n new facilities. If for any $r' < r$, the number of new facilities is more than n , then the solution to the covering problem, with covering radius r , is the solution to

the n -center problem. The covering problem can be modeled as an integer program and has received considerable attention in the Operations Research literature. Although it belongs to the class of NP-hard problems, special algorithms to solve reasonably sized problems are available (Fisher and Kedia, 1990). In addition, general techniques for solving integer programs can be used to solve small-sized problems.

On a tree network, the covering problem can be solved in linear time. This, along with the fact that the optimal covering radius for the n -center problem will be equal to $w_i w_j d(P_i, P_j) / (w_i + w_j)$ for some i and j , results in an efficient solution procedure for the n -center problem on a tree network.

n-Median Problem

Hakimi (1964) was the first to show that the set of all nodes of the network is a FDS for the n -median problem. Thus, for the 1-median problem, evaluating the objective function at each of the n nodes can identify an optimal location. As in the case of the 1-center problem, when the network is a tree, the problem is a convex problem and more efficient solution procedures are available. The "Chinese algorithm" given by Hua Lo-Keng and others (1962) and the "Majority algorithm" of Goldman (1971) work directly with the tree. If W denotes the sum of all the weights, the majority algorithm proceeds by choosing a tip node i of the tree with weight w_i . If $w_i \geq W/2$, then the tip is the solution; otherwise w_i is added to the weight of the adjacent node, node i and the edge adjacent to it are deleted and the procedure is repeated.

The n -median problem on a general network is difficult to solve, even though the candidate location set is only the set of nodes. As alluded to in the introduction, choosing n sites from a set of k ($k \geq n$) possible sites can be done in $k! / (n!(k-n)!)$ ways. Clearly this number can be very large. Thus, the approaches so far have been to formulate it as an integer program and to use integer programming methods.

Multifacility Minisum Problem

For the multifacility problem, an FDS is the set of nodes. The problem is often modeled as a mixed integer program for the case of a general network. For a tree network, efficient algorithms exist. One such algorithm (Kolen, 1986) starts by locating all new facilities at a tip of the tree, say node i , and then (by solving a max-flow problem) finding a subset S of the new facilities which when moved to the adjacent vertex will result in maximum improvement in the objective function value. If no such subset exists, then the algorithm stops. Otherwise a new problem is formed by considering S as the set

of new facilities to be located, the tree is modified by deleting node i and the adjacent arc and the weights are modified appropriately. This algorithm can also be used to solve the planar version of the problem with rectilinear distance, as discussed earlier.

In this problem, we can define a *flow graph* to represent the interaction between new facilities. For each new facility there is a node in the flow graph and when there is a nonzero interaction between a pair of new facilities, the two nodes in the flow graph are adjacent. Chhajed and Lowe (1992) gave an algorithm that exploits the structure of the flow graph for special cases.

Quadratic Assignment Problem

In the network version of the QAP, the existing facilities are located on the network and one new facility is to be assigned to each existing facility. There is no known polynomial time algorithm for the case of a general network. When the flow graph is a tree, Christofides and Benavent (1989) have given an algorithm that uses branch and bound (see section below) to solve a problem of size up to 25.

2.3.3 Mixed Integer Programming Models

Many of the Network models and some planar models can also be modeled as mixed integer programming (MIP) or pure integer programming (IP) problems. In an MIP, there are both integer and continuous variables whereas, in an IP there are only integer variables. For our purposes, we will not distinguish between them and will use MIP to refer to both. Most of our problems can be modeled using discrete variables which are special integer variables. A discrete variable can take on only one of two values (0 or 1) whereas, an integer variable can take on any non-negative integer value. The discrete variables are used to model either/or types of decisions (there are other uses for discrete variables). For example, if P_i is a candidate location site then a variable y_i can be defined to take a value 1 if a facility is located at P_i and 0 otherwise. If f_i is the fixed cost of locating a facility at P_i then the term $f_i y_i$ can be used to capture the fixed cost of a facility at i that will be incurred in a solution. Such an explicit consideration of fixed cost is not possible in planar or network models. MIP offers other flexibilities in incorporating a number of other real world considerations in the model. For example, there may be limitations on the capacity of a facility or there may be budget constraints.

Although a problem with many realistic constraints can be modeled well as an MIP, solving such a model is another matter. The technology in this regard has advanced considerably in the past two decades, but it is still limited. A number of computer codes are available to solve a general MIP. Examples are CPLEX, LINDO, OSL, SAS/OR, and

SCICONIC. (See Sharda (1992) for a survey of linear programming packages many of which also have MIP solving capabilities.) While these codes may be satisfactory for small sized problems, special techniques are required to solve large problems. Many applications also require the problem to be solved several times to perform "What-If" analyses. Thus, reasonably fast solution procedures are needed. Other issues in dealing with an MIP are data requirements and availability, complexity of using such methods, and the flexibility (or lack of it) offered by them. We next discuss some of the basic solution methods to solve an MIP.

There are three basic approaches we will discuss that are widely used: (i) explicit enumeration, (ii) implicit enumeration, and (iii) relaxation.

Explicit Enumeration: In explicit enumeration, all possible solutions are generated in a systematic way. An enumeration tree can be used to conceptualize the process (Figure 1). We start with a root node and create two arcs representing the two possible values of the variable y_1 . The two nodes generated are at level 1. At each level 1 node, we again create two arcs for two possible values of variable y_2 . The new nodes created are at level 2. This process continues for each variable in a sequence. A node at level i represents the solution where a decision has been made for variables $y_1, y_2, \dots, y_i$. The path from each end node to the root node represents a solution. The objective function value is computed for each solution and the optimum solution is selected.

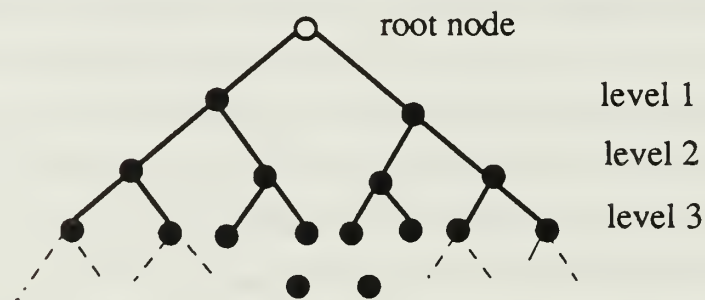


Figure 1. An Enumeration Tree

Implicit Enumeration: The explicit enumeration method is practical for only small problems. As the number of discrete variables grows, the complexity of enumeration, in terms of time and memory required, grows so fast that no matter how generous one's computer budget might be, it will soon not be enough to solve larger problems. This is because every time the number of variables is increased by one, the computational requirements roughly double. Thus, if it took 2 hours to solve a problem with 10

variables, it will take 64 hours to solve the same problem with 15 variables and 2048 hours to solve it with 20 variables. Implicit enumeration, also known as *branch and bound*, works on the principle of divide and conquer. It first divides the solution space (the set of all possible solutions). Each subspace is further divided into smaller subspaces until it can no longer be divided or it can be shown that a particular subspace can be pruned (omitted) because (i) no solution in it is feasible, or (ii) no solution in it will produce an optimal solution or (iii) further examination of the subspace is unnecessary because an optimal solution in the subspace is available.

Relaxation Approach: A relaxation based approach is used to generate a lower bound for the (minimization) problem. Such relaxations are also used in implicit enumeration to prune a subspace by the second criterion above. The most common relaxation is the linear programming relaxation in which the discrete variables are replaced by continuous variables that are constrained to be between 0 and 1. For many problems, this relaxation will give an integer solution, in which case it is also an optimal solution. Otherwise, a feasible solution can be generated by carefully rounding the variables. A *Lagrangian relaxation* is obtained by including some constraints in the objective function along with (Lagrange) multipliers, where they become "penalties" of a sort. In such an approach, care has to be taken in the choice of constraints to be taken to the objective function. The idea is that after the relaxation, the problem should be easier to solve, but at the same time, should be able to give good bounds. The choice of multipliers is tricky. Usually an optimization over the multipliers is performed to obtain the best possible bound (for a particular choice of relaxed constraints). This latter problem is called the Lagrangian dual problem. A "user friendly" description of this powerful approach is given in Fisher (1985). For more technical information, the reader is referred to Fisher (1981) and Geoffrion (1974).

We now discuss specific contributions for selected models discussed in section 2.2.

The Covering Problem

A number of logical tests (Fisher and Kedia, 1990) are proposed for the covering problem to test feasibility and to reduce problem size. These tests help in setting the values of some of the 0-1 variables before solving the problem. As an example (refer to model 2), if there is only one site within the required distance from a customer, then that site has to be open. Thus the corresponding x_j variable can be set to 1. This site may cover additional customers and consequently the customers this site covers can be

removed from the problem.

A number of implicit enumeration algorithms have been proposed. Surveys of these methods appear in Garfinkel and Nemhauser (1972) and Christofides and Korman (1975). Lagrangian relaxation-based approaches are given by Balas and Ho (1980) and Beasley (1987). Fisher and Kedia (1990) gave a dual heuristic which outperforms the previous best known algorithm of Balas and Ho. Their method solves the dual of the linear programming relaxation to provide a lower bound which is then used in a branch and bound algorithm. They report solving problems with 100-200 rows and 100-2000 columns.

n-Median Problem

Lagrangian relaxation-based procedures given by Cornuejols, Fisher and Nemhauser (1977) and Narula, Ogbu and Samuelsson (1977) appear to be fairly successful in solving the n-median problem. Galvao (1980) has given a dual-based approach which is similar to the Erlenkotter's dual heuristic for the simple plant location problem discussed in the next section. Another dual approach is given by Mavrides (1979). A thorough discussion of research contributions on the n-median problem and its generalizations can be found in Mirchandani (1990).

Quadratic Assignment Problem

A good algorithm for solving the quadratic assignment problem (QAP) has eluded researchers for several decades. Even finding an approximate solution to the problem within a specified percentage from the optimal value is a difficult problem (NP-hard). The QAP includes the well-known linear assignment problem and the traveling salesman problem as special cases.

Implicit enumeration algorithms have been proposed by Gavett and Plyter (1966), Lawler (1962), Gilmore (1962), and Mirchandani and Obata (1979). These methods are not suitable for problems involving more than 20 facilities. A number of heuristic solution procedures are also given by several researchers. A survey of these methods is given in Burkard (1990).

An interesting and surprising result for a probabilistic version of the QAP shown by Burkard and Fincke (1985) is that the relative difference between the worst and the optimal solutions becomes arbitrarily small with a probability tending to 1 as the problem size tends to infinity.

3. The Simple Plant Location Problem

Earlier in this paper, we have discussed the lockbox problem and the warehouse location problem (WLP). These problems can involve capacities, e.g., upper limits on the number of facilities opened and/or upper limits on the number of existing facilities served from an open facility. In this section we discuss the simple plant location problem (SPLP), which is a simplified version of both the lockbox problem and WLP. We focus on SPLP due to the extensive history of its study by operations researchers. We will not attempt to give a complete and thorough history since such information is available in Krarup and Pruzan (1983) and Cornuejols, Nemhauser and Wolsey (1990). Instead, our attempt is to discuss what we feel are the major operations research contributions to the analysis and solution of the problem.

The SPLP can be stated and formulated as follows: With S as the set of candidate sites for facilities, denote the sites by $j = 1, \dots, n$, where $|S| = n$. Let $f_j > 0$ be the fixed cost of locating (opening) a facility at site j , $j = 1, \dots, n$. Customers (existing facilities) are indexed by $i = 1, \dots, m$. The unit cost of servicing customer i from a new facility located at site j is $C_{ij} \geq 0$, $i = 1, \dots, m$, $j = 1, \dots, n$. The SPLP is to locate (open) the optimal number of sites, and to determine the optimal servicing pattern for all customers. Optimal in this case means at total minimum cost. Letting y_j be 1 or 0 (site j is open / not open) and letting x_{ij} denote the fraction of customer i 's service provided by site j , we now have (SPLP).

$$\text{Min } \sum_j f_j y_j + \sum_i \sum_j C_{ij} x_{ij} \quad (1)$$

$$\text{Subject to: } \sum_j x_{ij} = 1 \quad (2)$$

$$y_j = 0 \text{ or } 1 \quad (3)$$

$$x_{ij} \geq 0, i = 1, \dots, m, j = 1, \dots, n, \quad (4)$$

$$\text{and } x_{ij} \leq y_j, i = 1, \dots, m, j = 1, \dots, n, \quad (5a)$$

$$\text{or } \sum_i x_{ij} \leq m y_j, i = 1, \dots, n. \quad (5b)$$

Expression (1) totals site costs and service costs. The requirement that each customer be completely served is assured by (2). Expression (3) prevents a fractional opening of a site, and (4) assures nonnegative service. The condition that service cannot be provided from an unopened facility is guaranteed by either (5a) or (5b), and thus either of these constraint sets can be present. Constraints (5a) ((5b)) are often called strong (weak) constraints, and both versions will be discussed subsequently.

There appear to be two main reasons why SPLP has received so much attention by operations researchers. First, SPLP and its variants have found numerous applications. Secondly, SPLP appears (at first glance) to be seductively easy to solve - if not to optimality, then certainly to near optimality. Such reasoning would proceed as follows: If one knew which sites (plants) were open, i.e., which y_j 's were equal to one, then an optimal servicing pattern can be found by assigning each customer to the open site for which the service cost is smallest. Furthermore, ties (for smallest C_{ij}) can be broken arbitrarily, and so SPLP has the "single assignment property" (see Krarup and Pruzan, 1983), i.e., there is an optimal solution to the problem where each customer's service is provided by exactly one (open) plant.

Early approaches to solving SPLP were heuristic in nature. Probably the best-known of these approaches is the "add-bump-shift" algorithm of Kuenne and Hamburger (1963). The procedure involves opening plants one at a time until a point is reached where overall costs increase if another plant is opened. At this point, "uneconomical" plants are dropped (closed), and opened plants are "moved" if such a move decreases the objective function. In the initial "adding" phase, plants are opened in a greedy manner whereby with a given partial solution (set of open plants), the next plant to open is the currently unopened plant which renders the greatest savings in total system cost when opened.

Manne (1964) used a Steepest Ascent One Point Move Algorithm on SPLP. The main idea of this heuristic is to represent a current solution (set of open and closed plants) by an n -vector of zeroes (closed) and ones (open). At each iteration, a new solution is created by changing exactly one entry in the n -vector to its complement, the choice dictated by the largest reduction in objective function value. The algorithm terminates when no one-point move will provide a lower cost solution.

Balinski and Wolfe (1963) are credited with the first attempt to solve SPLP to optimality, making use of Benders decomposition. The idea of this decomposition scheme is to separate the integer optimization (the y 's) from the continuous variable optimization (the x 's) in a master problem-subproblem format. At each iteration, the subproblem sends information to the master problem, where improving choices for the y variables are obtained (see Benders, 1962). Computational experience with this method is reported in Balinski (1965).

Effroymsen and Ray (1966) gave a branch and bound algorithm for solving SPLP. Branching (creating subsets of solutions) is accomplished by fixing some of the y variables to either zero or one. Lower bounds are created by solving the linear

programming relaxation of the weak formulation (5b) of SPLP. Although the linear programming relaxation is trivial to solve, the bounds created are often not very tight.

Khumawala (1972) also made use of the weak formulation, and tested eight different branching rules (choice of the y variable to fix at zero or one) in the Effroymsen and Ray algorithm.

Spielberg (1969a) also used the weak formulation and implemented a search algorithm for SPLP. The initial search origin in the algorithm is either with all plants closed or all plants open. The algorithm implemented a single branching scheme at each step by opening one plant or closing one plant. Search directions are guided by information obtained from solving the linear programming relaxation of SPLP. In Spielberg (1969b), other search origins for the algorithm are considered.

D. Erlenkotter, and O. Bilde and J. Krarup independently developed a powerful dual-based method for solving SPLP. The methods are reported in Erlenkotter (1978), and Bilde and Krarup (1977). An interesting account of these efforts appears in Krarup and Pruzan (1983), and in Erlenkotter (1992). Both procedures make use of the linear programming relaxation of the strong formulation (5a) of SPLP, and near-optimal solutions to the dual linear program are found via an ascent heuristic. Erlenkotter's algorithm, called DUALOC, also incorporates a "dual adjustment" scheme (to improve the dual solution found in the ascent phase) as well as a primal (original problem) recovery procedure. DUALOC appears to be a very effective method for SPLP. Erlenkotter (1978) reports solving to optimality 45 of the 48 problems he tested. Evidence of the acceptance of DUALOC as a primary method of choice for solving SPLP is provided by Erkut (1992), who reports, as of 1991, 119 citations to Erlenkotter's paper in the Social Sciences Citation Index.

Inspired by the success of DUALOC, Cornuejols and Thizy (1982) use a primal subgradient algorithm to solve the linear programming relaxation of the strong formulation of SPLP. The subgradient procedure is embedded in a heuristic where, after a certain number of subgradient iterations, some of the y variables have converged to zero or one. These variables are then fixed at the appropriate value and a smaller location problem is derived. This smaller problem is then often solved with a greedy interchange heuristic.

Most of the work that we have cited above involves heuristics to find "good" solutions to SPLP. The main disadvantage of any heuristic is that one often has no idea of how "good" a particular heuristically-derived solution might be, relative to the optimal solution of the problem. Several authors have studied the "quality" of various heuristics applied to SPLP and its variants. From a worst-case standpoint, an excellent reference is

Cornuejols, Fisher and Nemhauser (1977). Average-case analysis is considered in Cornuejols, Nemhauser and Wolsey (1980) and Fisher and Hochbaum (1980).

Another line of research on SPLP involves the study of the set of feasible solutions to (2) through (5a). Letting F denote the set of feasible solutions, then any vector $[X, Y]$ in F has the property that each element y_j of Y is either zero or one. Let F_R denote the set of feasible solutions to the linear programming relaxation of SPLP (with 5a)). Thus F_R contains "fractional" vectors $[X, Y]$ where one or more entries of Y are fractional. A previously mentioned method of solving SPLP is via branch and bound, where lower bounds are computed via linear programming relaxation. In an effort to strengthen the lower bounds, several researchers have attempted to identify linear inequalities (involving X and Y) which when appended to the linear programming relaxation of SPLP, will eliminate "fractional" optimal solutions to the linear program.

As an example of this work, Cho, Johnson, Padberg and Rao (1983a, 1983b) identify such inequalities for versions of SPLP where either $n=3$ and $m \geq 3$, or $m = 3$ and $n \geq 3$. (The cases with $m \leq 2$ and $n \leq 2$ are easily solved.)

Dearing, Hammer and Simeone (1992) give a pseudo-Boolean formulation of SPLP, and then transform this formulation into both a set covering problem, as well as a vertex packing problem on a problem-specific graph, which they call the "plant location graph."

Krarup and Bilde (1977) have characterized a class of SPLP's which are solvable in polynomial time, and provide a solution algorithm for such instances. The characterization involves dominance properties of the service cost coefficients (the C_{ij} 's). Kolen (1983) has shown that SPLP is polynomially solvable when the C_{ij} 's are derived from weighted distances on tree networks. The algorithms provided for these problems are specific implementations of the dual ascent procedure mentioned earlier.

There is a strong connection between SPLP and the covering problem (Model 2. discussed earlier) in the following sense: Krarup and Pruzan (1983) have shown that the covering problem can be transformed to an equivalent SPLP; and A. Tamir (see Kolen, 1983) has demonstrated that SPLP can be transformed to an equivalent covering problem. Hoffman, Kolen and Sakarovich (1985) give a polynomial time algorithm for solving the covering problem when the covering matrix has certain properties. Since the SPLP and the covering problem are essentially equivalent (due to the transformation results), Xu and Lowe (1992) study the connection between the dual ascent procedure for SPLP and the algorithm of Hoffman, Kolen and Sakarovich for the covering problem. As a by-product they identify a class of SPLP's which contain the Krarup and Bilde, and Kolen problems, and are polynomially solvable via dual ascent.

4. Summary

We have considered three different classes of models: planar models, network models, and MIP models. Table 1 summarizes these three classes, and compares them in terms of four important attributes: realism, data requirements, computational requirements, and difficulty of explanation/lack of insight. Note the price paid for realism is in the last three attributes. This means there will probably continue to be a place for planar models, particularly when limited resources are involved. It also means the choice of an appropriate model will probably continue to be problem-dependent, data-dependent, and even computer-dependent.

TABLE 1: ATTRIBUTE RANKINGS FOR THE THREE MODEL CLASSES

Key: L: Low M: Medium H: High

Attributes \ Classes	Planar	Network	MIP
Realism	L	M to H	M to H
Data Requirements	L	M to H	H
Computational Requirements	L	M	H
Difficulty of Explanation/ Lack of Insight	L	M	H

Also we think it is important for Operations Researchers to note that many people outside their profession who would like to use their results may be reluctant to do so, because they do not understand the methodology, or because they have little or no insight into why the answers provided are correct. Indeed, the table identifies some topics deserving more attention.

We hope researchers will consider developing planar models, when possible, together with network and/or MIP models, in order to help provide insight for users. See Geoffrion (1976) for a good discussion, and an example, of using planar models in conjunction with an MIP model, for exactly this purpose.

More attention deserves to be given to data considerations in developing location models. According to folklore, getting the data is 90% of solving a location problem, and we have no reason to disagree with this estimate. In spite of this, there is a tendency to be preoccupied with getting optimal solutions, assuming all the data is available and given, and that sensitivity analysis is of little importance. We think researchers should direct more attention to data aspects in order to expedite the use of their work. For example, in urban and regional location contexts there may well be more than 100,000 existing facilities, in which case one has little choice but to aggregate existing facilities into some smaller collection. Yet the amount of literature available addressing the theory for doing aggregation is relatively small. As a related issue, does it really make sense to find an optimal solution to an aggregated problem, which may be only a crude approximation to the actual problem? Similarly, we believe a familiarity with census population data bases, and street network data bases, might well spur useful research on the design of models which can interface well with available data. Perhaps also the sheer size of many realistic location problems would suggest that more attention should be paid to heuristic algorithms, since they typically are computationally less intensive than algorithms which find optimal solutions.

In view of the fact that many varieties of models may well be available for solving what amounts to the same problem, e. g., planar, network or MIP n -median problems, there is a need for something like an expert system to help the layperson choose the model best suited for his/her purposes.

Apart from the availability of commercial packages for solving LP, MIP, and network optimization problems, such as IBM's OSL package, finding appropriate software for a particular algorithm is likely to be a "hit or miss" situation. Currently, one has little option but to contact the authors of the algorithms of interest and hope they will agree to provide the software. We think the profession should give serious consideration to encouraging the development of software systems for the algorithms in greatest use. Until this is done, it will continue to be the case that reliance must be placed upon astute consultants who have taken the trouble to develop their own software. Probably most readers who use a word processor can verify from their own experience that they did not bother to do so until software developers made it very easy for them to do so. Why should the situation be any different for potential users of location algorithms? Can anybody claim that today's algorithms are easy to use by laypersons?

As regards solving MIP models, there will always be a place for algorithms that can solve larger problems faster and better. Much the same conclusion applies to network models.

Many location problems have a substantial stochastic aspect, and may well have time-dependent aspects as well. While there has been some very good work to address such aspects, the total amount of it is relatively small by comparison to the amount of work in math-programming oriented models. We hope efforts will be made to remedy this situation in the future.

In conclusion, we think it might be useful for researchers to try to take a "systems view" to the solution of location problems. They could ask "what does the user need?" instead of "what can we provide?". Answers to the former question might well lead to new and interesting research endeavors, and at the same time increase the economic benefit of such activities.

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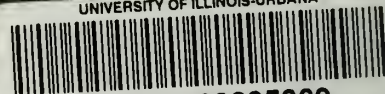
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